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$$\frac{1}{\sqrt{1-n^2}} \log \frac{n \tan \frac{1}{2} \omega - 1 - \sqrt{1-n^2}}{n \tan \frac{1}{2} \omega - 1 + \sqrt{1-n^2}} = \varphi + c_2.$$

$$\therefore \frac{1}{\sqrt{1-n^2}} \log \frac{n \tan \left( \frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right) - 1 - \sqrt{1-n^2}}{n \tan \left( \frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right) - 1 + \sqrt{1-n^2}} = \tan^{-1} y + c_2.$$

If  $n=1$ ,  $\frac{d\omega}{1-\sin\omega} - d\varphi = 0$ .

$$\therefore \frac{d\omega}{\sin^2 \frac{1}{2} \omega + \cos^2 \frac{1}{2} \omega - 2 \sin \frac{1}{2} \omega \cos \frac{1}{2} \omega} - d\varphi = 0.$$

$$\therefore \frac{d\omega \sec^2 \frac{1}{2} \omega}{(\tan \frac{1}{2} \omega - 1)^2} - d\varphi = 0. \quad \therefore \frac{2d(\tan \frac{1}{2} \omega - 1)}{(\tan \frac{1}{2} \omega - 1)^2} - d\varphi = 0.$$

$$\therefore \frac{2}{1 - \tan \frac{1}{2} \omega} = \varphi + c_2. \quad \therefore 1 + \frac{\tan \frac{1}{2} \omega + 1}{1 - \tan \frac{1}{2} \omega} = \varphi + c_2.$$

$$\therefore \tan \left( \frac{1}{2} \omega + \frac{1}{4} \pi \right) = \varphi + c'^2.$$

$$\therefore \tan \left[ \frac{\pi}{4} + \frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right] = \tan^{-1} y + c'_2; \text{ if } n = -1,$$

$$\frac{d\omega}{1 + \sin \omega} - d\varphi = 0, \text{ and the result is } \tan \left[ \frac{\pi}{4} - \frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right] = \tan^{-1} y + c.$$

Also solved by *L. C. WALKER*, *H. C. WHITAKER*, and *G. B. M. ZERR*.

123. Prize Problem. Proposed by *B. F. FINKEL*, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Find in finite terms, the value of  $\int_0^{\frac{1}{2}\pi} \log \tan \phi d\phi$ .

Solution by *W. H. ECHOLS*, Professor of Mathematics, University of Virginia, Charlottesville, Va.

$$\text{Put } \tan z = x. \quad \text{Then } \int_0^{\frac{1}{2}\pi} \log(\tan z) dz = \int_0^1 \frac{\log x}{1+x^2} dx,$$

$$= \int_0^1 \log x (1 - x^2 + x^4 - x^6 + \dots) dx.$$

$$\text{Differentiating, respecting } a, \quad \int_0^1 x^a dx = \frac{1}{a+1}.$$

$$\therefore \int_0^1 x^a \log x dx = -\frac{1}{(a+1)^2}.$$

$$\therefore \int_0^{4\pi} \log(\tan z) dz = -\left(1 - \frac{1}{3^2} + \frac{1}{5^2} - \dots\right) = -\frac{\pi^3}{32},$$

by a well known summation.

A correct result was also received from Lon. C. Walker. The integration in finite terms required that the entire work should be done in finite terms as is done, for example, in Byerly's *Integral Calculus*, page 98, when

$$\int_0^{\frac{1}{2}\pi} \log \sin x dx$$

is found, and not the final result stated in finite terms.

The summation of the above series may be found by using the relation,

$$\frac{B_{2n}}{(2n)!} = \frac{2^{2n+2}}{\pi^{2n+1}} \left[ 1 - \frac{1}{3^{2n}} + \frac{1}{5^{2n}} - \frac{1}{7^{2n}} + \dots \right]$$

When  $n=1$ ,  $B_2=1$ , one of Euler's numbers. See B. O. Peirce's *Table of Integrals*, page 90. Ed.

124. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Show that the cardioids  $r=a(1+\cos\theta) \dots (1)$ , and  $r=b(1-\cos\theta) \dots (2)$ , intersect at right angles.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

The equation can be written  $r=2a\cos^2\frac{1}{2}\theta$ ,  $r=2b\sin^2\frac{1}{2}\theta$ , or  $r^{\frac{1}{2}}=(2a)^{\frac{1}{2}}\cos\frac{1}{2}\theta$ ,  $r^{\frac{1}{2}}=(2b)^{\frac{1}{2}}\cos\frac{1}{2}(\pi-\theta)$ .

$$\frac{dr}{rd\theta} = -\tan\frac{1}{2}\theta, \quad \frac{dr}{rd\theta} = +\tan\frac{1}{2}(\pi-\theta).$$

At the point of intersection  $r$  and  $\theta$  are the same for both.

The angle made by the perpendicular from the origin on the tangent is in the first case  $\frac{1}{2}\theta$ , in the second  $\frac{1}{2}(\pi-\theta)$ .

But  $\frac{1}{2}\theta + \frac{1}{2}(\pi-\theta) = \frac{1}{2}\pi$ .

$\therefore$  These perpendiculars are perpendicular to one another.

$\therefore$  The tangents are perpendicular and the cardioids intersect at right angles.

Solved substantially as above by JOHN F. TRAVIS, Fellow and Assistant in Mathematics in Ohio State University, Columbus, O.; E. L. SHERWOOD, A. M., Professor of Mathematics, Beaver College, Beaver, Pa., and L. C. WALKER.

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## MECHANICS.

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125. Proposed by T. U. TAYLOR, C. E., Professor of Civil Engineering, University of Texas, Austin, Tex.

(1) If a parabola is described on the vertical face of a reservoir wall, axis vertical and in the surface, and  $P(h, b)$  be any point on the curve, and  $B$  the foot of the perpendicular from  $P$  on the axis, find c. p. on area  $OBP$ .

(2) If  $A$  is point where horizontal through  $P$  cuts vertical axis ( $OY$ ), find c. p. on area  $OAP$ .